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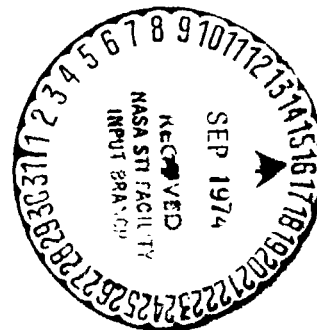
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## ATMOSPHERIC CONVECTIVE VELOCITIES AND THE FOURIER PHASE SPECTRUM

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## TABLE OF CONTENTS

	Page
INTRODUCTION . . . . .	1
BACKGROUND . . . . .	2
THEORETICAL ANALYSES . . . . .	3
DISCUSSION AND CONCLUSIONS . . . . .	12
REFERENCES . . . . .	13

## LIST OF ILLUSTRATIONS

Figure	Title	Page
1.	Convective velocity evaluation . . . . .	5
2.	Correlation measurement of convective velocities as a function of frequency. . . . .	6
3.	Correlation measurement of convective velocities . . . . .	7
4.	Convective velocity versus frequency . . . . .	11

## TECHNICAL MEMORANDUM X-64869

### ATMOSPHERIC CONVECTIVE VELOCITIES AND THE FOURIER PHASE SPECTRUM

#### INTRODUCTION

The equations of motion for the near-ground atmospheric boundary layer have proven difficult to solve in closed form. The need for a solution or simulation of the statistical behavior of the atmospheric boundary layer for aerospace and aeronautical applications is ever increasing with the engineering demands for sophistication in the modeling of wind loads for stress and harmonic analyses of spacecraft and conventional aircraft. Because solutions of the equations of motion for the atmospheric turbulent boundary layer have proven so elusive, it appears more fruitful to turn to boundary layer modeling or statistical simulation concepts.

The modeling or statistical simulation of the atmospheric boundary layer is controlled by the physical motion of the undulations or fluctuations of the velocity field. Organized fluid structures are responsible for fluctuations that deviate from the mean. The examination of organized fluid structures embedded within the boundary layer would give valuable insight into the understanding of the physical flow structure and present one piece of the puzzle necessary for the statistical simulation of our natural environment.

Organized fluid structures (sometimes referred to as "packets of vorticity" or, more loosely, as "eddies") which are embedded within the atmospheric boundary layer play a key role in the dynamics of the fluid motion. The velocity with which these structures move is termed "convective velocity." The relative motion of the organized structures with respect to the mean motion in regions of high shear is yet to be firmly understood, although a likely first assumption is that the mean motion is the forcing function driving the structures. The high shear region (where little is known concerning convective velocities) of the atmospheric boundary layer consists of approximately the first 150 meters [1].

The purpose of this report is to present a method by which a more intimate knowledge of the organized structure motion embedded in the atmospheric boundary layer may be obtained by employing a variation of the Fourier phase spectrum of the space-time correlation.

## BACKGROUND

In 1915 G. I. Taylor examined "eddy" motion in the atmosphere and made the first documented statement about the convective transport ability of the "eddy" when he said, "It seems natural to suppose that eddies will transfer not only the heat and water vapor, but also the momentum of the layer in which they originated to the layer with which they mix" [2]. Thus, if an eddy moves from a layer of low momentum to one of a higher momentum, one can imagine that the eddy will have a momentum different from the new layer until the eddy has been acted upon by the new layer for a sufficiently long dwell time for the eddy to come into equilibrium with the new layer. In 1917 Taylor performed experiments to map the shape of eddies in the atmosphere and found the shape to be dependent upon the elevation above the ground [3]. Taylor then went on in 1938 to publish the relationship between correlation measurements and spectra [4]. This study laid the foundation for future work using correlation measurements to describe particular motions of selected portions of the spectrum. It is here that Taylor gave insight into the motions of eddy structures as compared with mean fluid motions. He stated, "If the velocity of the air stream which carries the eddies is very much greater than the turbulent velocity, one may assume that the sequence of changes in  $u'$  at the fixed point are simply due to the passage of an unchanging pattern of turbulent motion over the point, i. e., one may assume that

$$u' = \phi(t) = \phi(X/V_m) \quad , \quad (7)$$

where  $X$  is measured upstream at time  $t = 0$  from the fixed point where  $u$  is measured. In the limit when  $u'/V_m \rightarrow 0$  (7) is certainly true. Assuming that (7) is still true when  $u'/V_m$  is small but not zero,  $R_x$  is defined as

$$R_x = \frac{\phi(t) \phi(t + X/V_m)}{\overline{u'^2}} \quad ."$$

This quote later became known as Taylor's hypothesis. Taylor's hypothesis has since been checked for grid-generated turbulence many times [5-9].

Since 1938, when Taylor first published what is now known as Taylor's hypothesis, the scientific community has attempted to use the hypothesis in

areas which do not rigidly conform to Taylor's original criteria. One such area is the atmospheric boundary layer. Here the turbulent velocity may not be small compared to the mean velocity and, thus, does not conform to the constraints of Taylor's hypothesis. In general, regions of high shear will have turbulent velocities which may not be considered small when compared to the mean velocity.

Measured convective velocities in wind tunnel boundary layer flows have proven to be, in general, different from the local mean velocity [16-21]. Atmospheric research in the area of convective velocities has been avoided in the past partly due to the physical complexities of obtaining space-time correlations in the field. Limited measurements in this area have been performed by Powell [22]. Powell's results indicate that Taylor's hypothesis is not valid for the atmospheric boundary layer near the earth's surface for frequencies below approximately 0.1 Hz, which is the spectral region containing the preponderance of energy. Similarly, Cliff and Sandborn [10] noted that the higher the frequency, the closer the convective velocity approached the local mean in wind tunnel boundary layers. If the eddies do not move at the same rate as the local mean flow, the transformation from time domain statistics to space domain statistics is not a linear relationship as expressed in Taylor's hypothesis. This means that transforming Eulerian time statistics to Eulerian space statistics using Taylor's hypothesis could lead to improper turbulent length scales as well as spectra. That is, a large, fast-moving structure and a smaller, slow-moving structure could theoretically produce the same time frequency.

## THEORETICAL ANALYSES

Convective velocities are most commonly measured by employing two velocity sensors spatially separated in the mean flow direction. The outputs of the two sensors are cross correlated with a varying time delay. A peak will occur in the cross correlation at the most probable time delay for which the signals are most similar. The convective velocity would then be equal to the separation distance divided by the most probable time delay. The cross correlation of the functions  $f(t)$  and  $g(t)$  would be expressed as:

$$C_{fg}(X, Y, Z, \xi, T, \tau) = \overline{f(X, Y, Z, t) g(X + \xi, Y, Z, t + \tau)} \quad (1)$$

For generality the coordinate system has been chosen such that the mean flow direction is the X-axis. The overbarred quantity is the time average over the sample record length  $T$ , which may be expressed as:

$$\overline{(\quad)} = \frac{1}{T} \int_0^T (\quad) dt \quad . \quad (2)$$

If the process is horizontally homogeneous and stationary and we note that  $Z$  and  $\xi$  are fixed (i. e. , constants) for this particular operation, the expression for the correlation function would be:

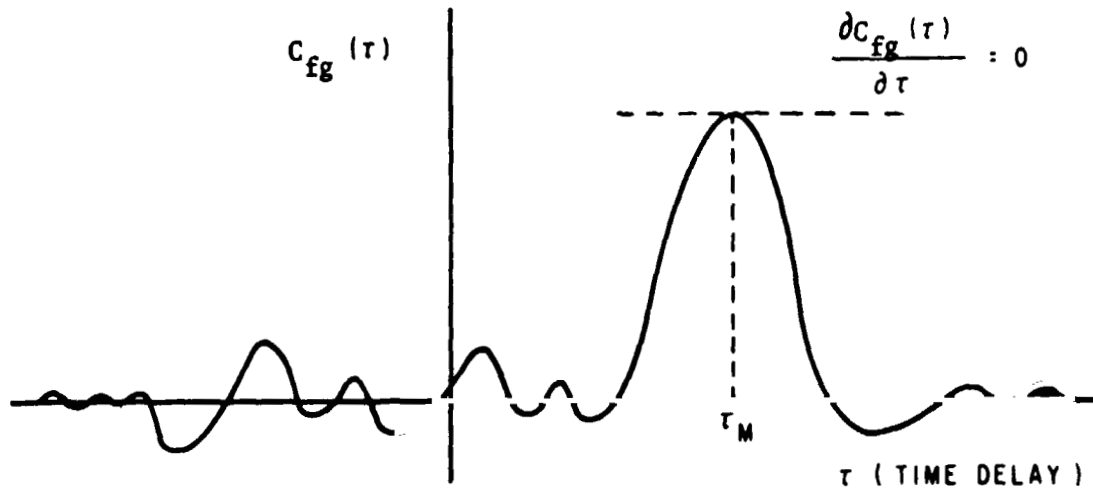
$$C_{fg}(\tau) = \overline{f(X, Y, Z, t) g(X + \xi, Y, Z, t + \tau)} \quad . \quad (3)$$

Figure 1 presents graphically what the cross correlation would look like and the method used to compute the convective velocity. (It should be noted that for the correlation concepts presented here one may or may not wish to have the means subtracted from the data. Correlation with the means not removed merely shifts the abscissa up or down and does not affect the location of the optimum time delay necessary for the computation of the convective velocity. )

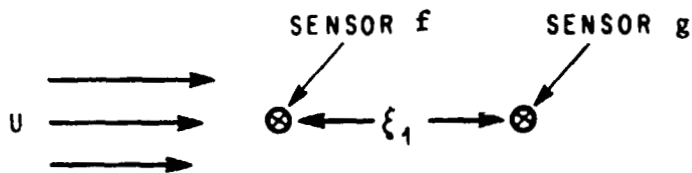
It should also be mentioned that one may use a fixed time delay and vary the separation distance to locate an optimum separation length rather than an optimum time delay. The convective velocity is then evaluated by dividing the optimum separation distance by the selected time delay. The differences between the fixed separation and fixed time delay methods of computing the convective velocity are discussed in detail by Cliff and Sandborn [10].

If one desires to look at the convective velocity of an individual frequency (frequency being a function of size and the velocity with which the fluid structure is swept past the Eulerian sensor), the correlating functions could be filtered for that particular frequency and the regular rules for evaluating the convective velocity would apply. If the measurement is performed in a high shear region where Taylor's hypothesis is not valid, the convective velocity could become a function of frequency. Figure 2 is such a case obtained in a flat plate wind tunnel boundary layer [10]. Note the shifting optimum time delay as a function of frequency. The optimum time delay shifting indicates that the convective velocity is a function of frequency. Thus, if one can relate size and frequency, the convective velocity then becomes a function of size.

In a region where Taylor's hypothesis could be considered valid, all harmonics (frequencies) would move at the same velocity and the optimum time delay would be independent of frequency (i. e. , a constant equal to the separation distance divided by the mean velocity of the flow). Figure 3 is an example of



(a) CORRELATION



(b) PROBE POSITION

$$C_v = \text{CONVECTIVE VELOCITY} = \xi_1 / \tau_M$$

Figure 1. Convective velocity evaluation.

such a case, which was also found in a wind tunnel after the fluid structures had been imbedded in the flow for a sufficiently long dwell time for the structures to come into equilibrium with the mean motion in the outer portion of the boundary layer [10]. Note that the optimum time delay is independent of the frequency at which the signals were filtered.

Rather than filtering at each frequency, the convective velocity may be obtained by examining the phase shifts of the individual harmonics between the velocity sensed upstream and the velocity sensed downstream. The phase analysis is performed by considering the Fourier representations of the two time functions used for the cross correlation,  $f(t)$  and  $g(t)$ . Since in practice



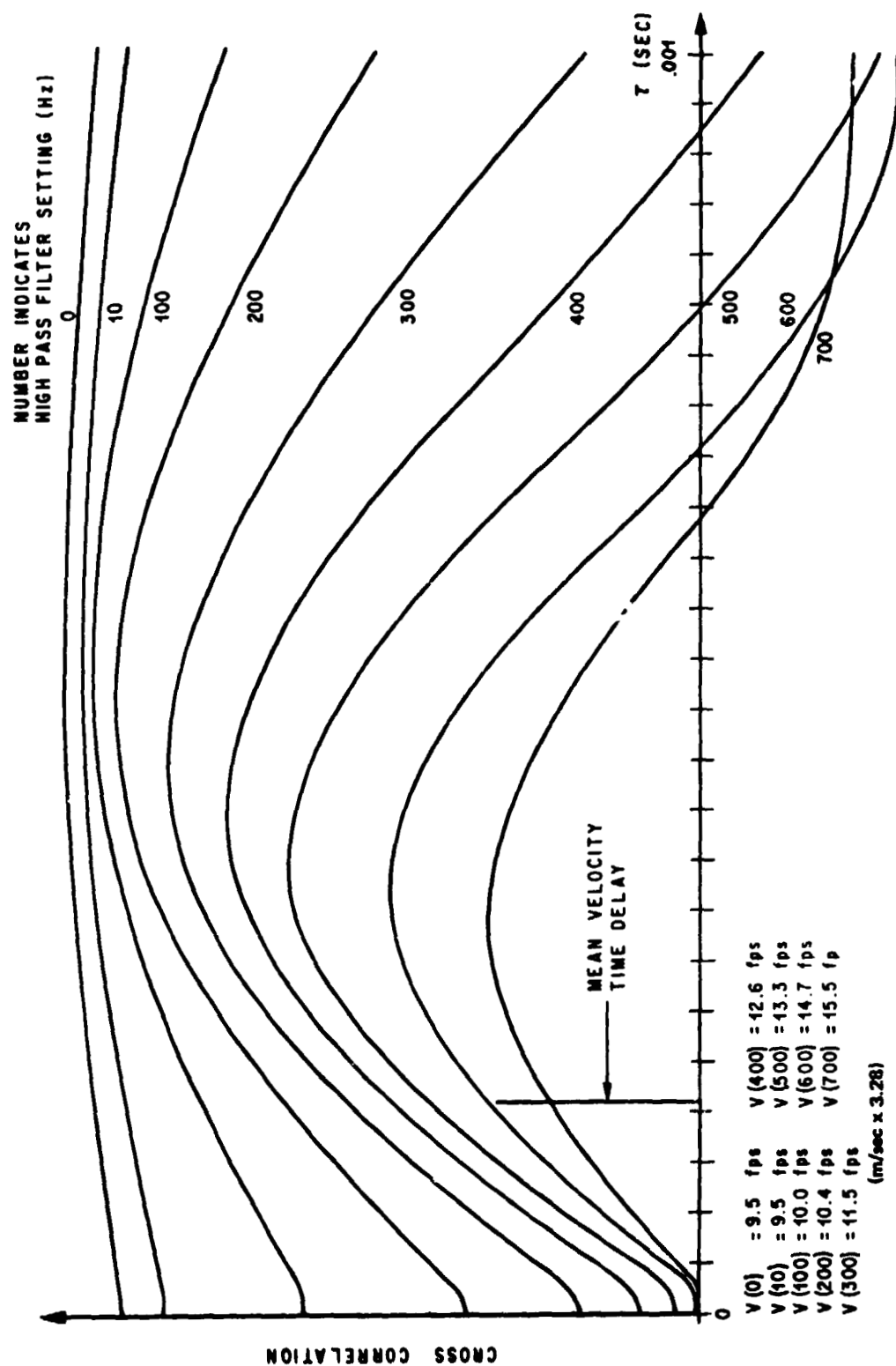


Figure 2. Correlation measurement of convective velocities as a function of frequency.

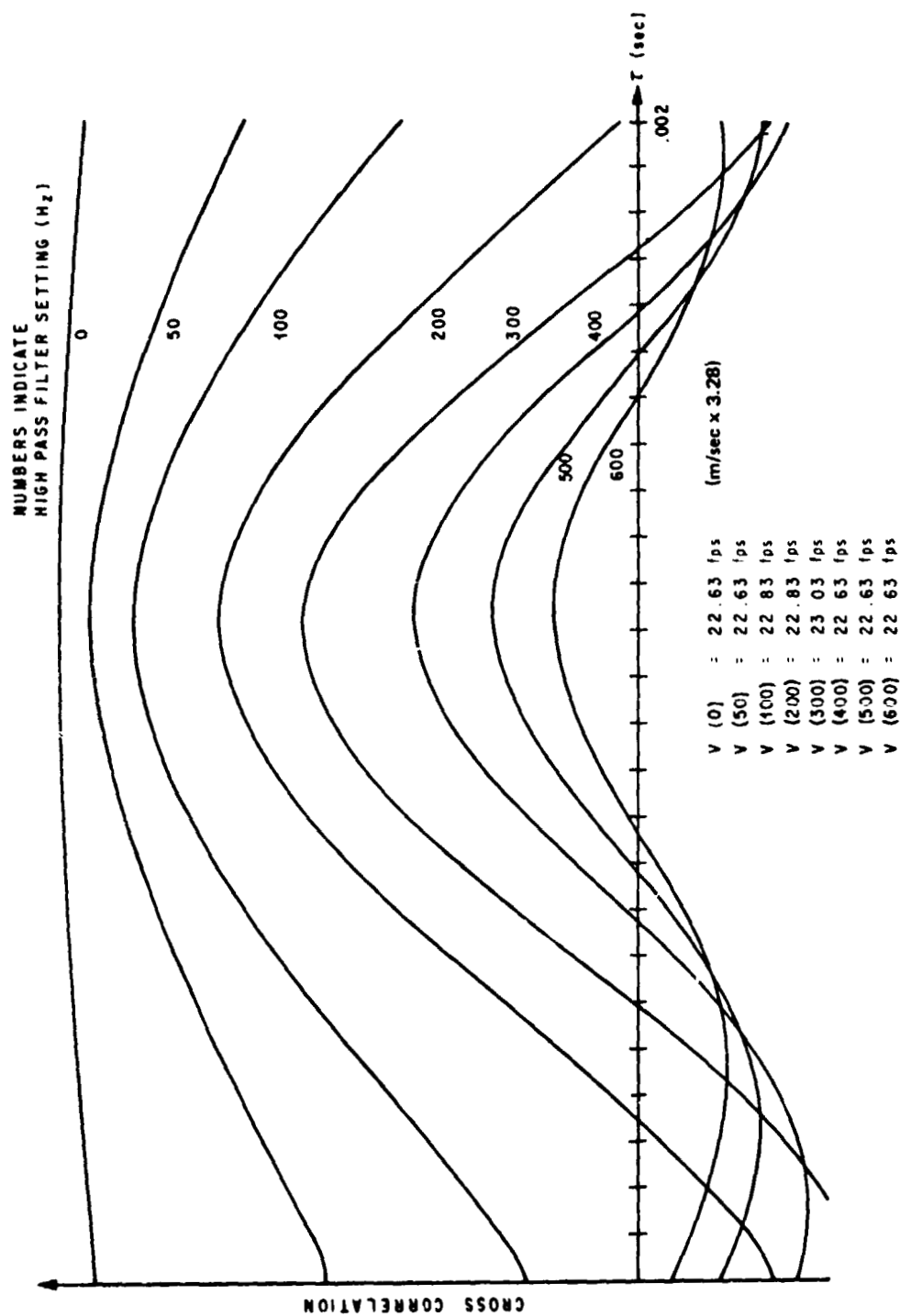


Figure 3. Correlation measurement of convective velocities.

we generally deal with data records of finite length, we shall use the Fourier series representation of the functions rather than the integral forms.

Consider the two time functions and their respective Fourier transforms:

$$\begin{aligned} f(t) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n t}{T} + b_n \sin \frac{2\pi n t}{T} \\ &= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)^{1/2} \cos\left(\frac{2\pi n t}{T} - \theta_{f,n}\right) \end{aligned} \quad (4)$$

and

$$\begin{aligned} g(t) &= \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos \frac{2\pi n t}{T} + d_n \sin \frac{2\pi n t}{T} \\ &= \frac{c_0}{2} + \sum_{n=1}^{\infty} (c_n^2 + d_n^2)^{1/2} \cos\left(\frac{2\pi n t}{T} - \theta_{g,n}\right) \end{aligned} \quad (5)$$

where  $\theta_{f,n}$  and  $\theta_{g,n}$  represent the phase shift of the  $n$ th harmonic at the origin and  $\theta_{f,n} = \tan^{-1}(b_n/a_n)$  and  $\theta_{g,n} = \tan^{-1}(d_n/c_n)$ .

The correlation of the functions  $f(t)$  and  $g(t)$  then becomes:

$$\begin{aligned} C_{fg}(\tau) &= \frac{1}{T} \int_0^T f(t) g(t+\tau) dt = \frac{a_0 c_0}{4} + \frac{1}{2} \sum_{n=1}^{\infty} \left[ a_n c_n \cos \frac{2\pi n \tau}{T} \right. \\ &\quad \left. + a_n d_n \sin \frac{2\pi n \tau}{T} - b_n c_n \sin \frac{2\pi n \tau}{T} + b_n d_n \cos \frac{2\pi n \tau}{T} \right] \end{aligned} \quad (6)$$

Since the correlation function may be thought of as a function itself without regard to the processes which created the correlation, one may write the correlation in a general Fourier expansion of its own as:

$$C_{fg}(r) = \frac{e_0}{2} + \sum_{n=1}^{\infty} e_n \cos \frac{2\pi nr}{T} + f_n \sin \frac{2\pi nr}{T} \quad (7)$$

$$= \frac{e_0}{2} + \sum_{n=1}^{\infty} (e_n^2 + f_n^2)^{1/2} \cos\left(\frac{2\pi nr}{T} - \theta_{c,n}\right) ,$$

where  $\theta_{c,n}$  is the phase shift from the origin of the nth harmonic and  $\theta_{c,n} = \tan^{-1}(f_n/e_n)$ . The values of the original time functions which represent  $f_n$  and  $e_n$  may now be taken from equation (6), yielding an expression for  $\theta_{c,n}$  in terms of the phase shifts of the original time functions.

$$\theta_{c,n} = \tan^{-1} \frac{f_n}{e_n} = \tan^{-1} \frac{a_n d_n - b_n c_n}{a_n c_n + b_n d_n} = \tan^{-1} \frac{\frac{d_n}{c_n} - \frac{b_n}{a_n}}{1 + \frac{b_n}{a_n} \frac{d_n}{c_n}} \quad (8)$$

$$= \tan^{-1} \frac{\tan \theta_{g,n} - \tan \theta_{f,n}}{1 + \tan \theta_{g,n} \tan \theta_{f,n}} = \tan^{-1} [\tan (\theta_{g,n} - \theta_{f,n})] .$$

Thus,

$$\theta_{c,n} = \theta_{g,n} - \theta_{f,n} \quad (9)$$

The interpretation of equation (9) is as follows: "The phase of the nth harmonic of the correlation is equal to the difference of the starting phases of the nth harmonics of the original time series from which the correlation was derived."

A special case results when  $f(t)$  equals  $g(t)$ ; that is, the correlation function becomes the autocorrelation function. For this case  $\theta_{g,n}$  would be identical with  $\theta_{f,n}$ , and thus  $\theta_{c,n}$  would be identically zero for all harmonics. That the phase spectrum of the autocorrelation is everywhere equal to zero is well known and may also be derived by considering the evenness of the autocorrelation.

The correlation receives its major contribution from the  $n$ th harmonic when

$$\cos \left( \frac{2\pi n\tau}{T} - \theta_{c,n} \right) = 1 \quad ; \quad (11)$$

that is, when

$$\theta_{c,n} = \frac{2\pi n\tau}{T} = 2\pi f\tau \quad . \quad (12)$$

If all of the waves traveled such that they peaked at approximately the same time delay, the correlation curve would have a pronounced maximum. The optimum time delay for the  $n$ th harmonic can now be calculated from equation (11):

$$\tau_m = \frac{\theta_{c,n}}{2\pi f} \quad . \quad (13)$$

And, finally, the convective velocity may be written as a function of the phase spectrum of the cross correlation. Noting the method for determining the convective velocity as being the separation distance divided by the optimum time delay, the convective velocity becomes:

$$V_{c,n} = 2\pi f\xi / \theta_{c,n} \quad , \quad (14)$$

where  $V_{c,n}$  is the convective velocity of the  $n$ th harmonic.

Equation (14) now gives a means to plot the convective velocity from the Fourier phase spectrum of the cross correlation. Figure 4 gives a graphical picture of what equation (14) plotted against frequency should be for Taylor's

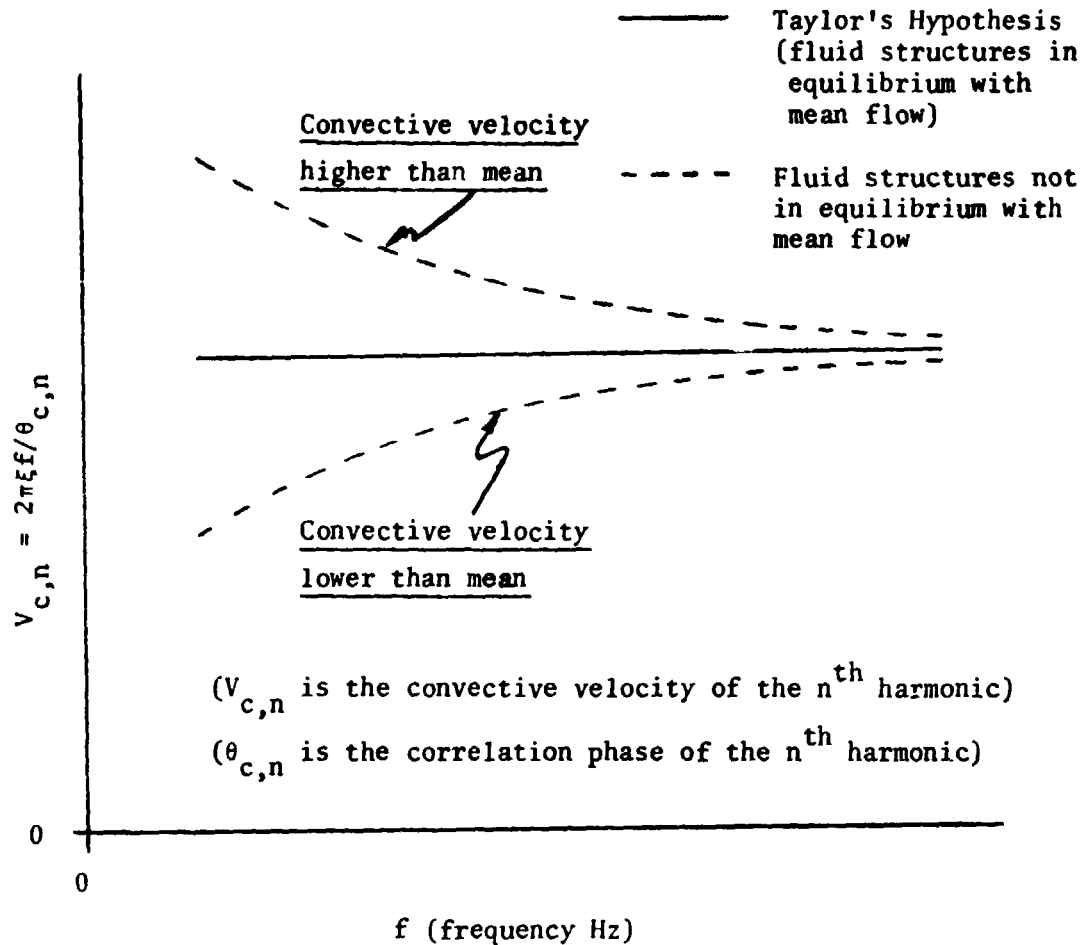


Figure 4. Convective velocity versus frequency.

hypothesis to be valid and to be invalid. The case of Taylor's hypothesis being invalid is represented by dashed lines in Figure 4. The top dashed line indicates that fluid structures are migrating from a region of higher momentum to a region of lower momentum. The lower dashed line indicates that the fluid structures are migrating from a region of lower momentum to a region of higher momentum. The shapes of the dashed lines are indicative of Cliff and Sandborn's model [10] predicting that the higher the frequency sampled, the nearer the convective velocity is to the mean. This seems reasonable since in turbulent shear layers the amplitude of the spectrum decreases with increasing frequency such that the limit as  $f \rightarrow \infty$ ,  $a_n \rightarrow 0$ , where  $a_n$  is the Fourier amplitude of the  $n$ th harmonic.

That is to say, one may wish to hypothesize that even in high shear regions

Taylor's hypothesis may be approached when the Fourier amplitude of the frequency sampled becomes much, much smaller than the mean velocity. The level line of Figure 4 represents the case where Taylor's hypothesis holds. Here all harmonics convect at the same velocity, which is very nearly the mean velocity.

## DISCUSSION AND CONCLUSIONS

A method of calculating the convective velocity employing a variation of the Fourier phase spectrum is developed. The convective velocity spectrum is shown to yield information concerning the equilibrium of the fluid structures and the mean flow.

If fluid structures are in equilibrium with the surrounding fluid, no acceleration or deceleration of the structure (eddy) would be expected and the structure should move with the mean motion of the fluid within which the structure is embedded. If the fluid structure is not in equilibrium with the surrounding fluid (such as an eddy propagating from a region of lower momentum to a region of higher momentum), one would suspect that the fluid structure would not be moving with the mean motion of the new layer to which it migrated until the structure had been acted upon by the new layer for a sufficiently long dwell time. For the latter case the convective velocity of the structure (eddy) would be different from the local mean velocity. Thus, transforming time statistics to space statistics for the second case would be misleading if Taylor's hypothesis was used. It now becomes evident that more work needs to be performed in the area of convective velocities in the high shear regions of the atmospheric boundary layer if one wishes to convert time statistics to space statistics in these shear regions, such as the lower portions of the atmosphere.

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APPROVAL

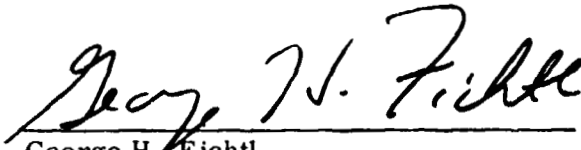
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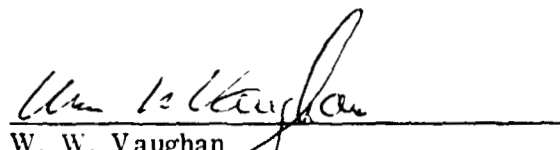
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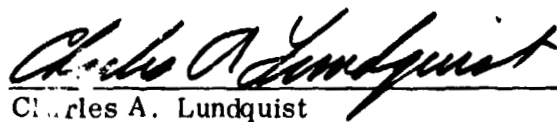
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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

  
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